3차원 Viewing 변환

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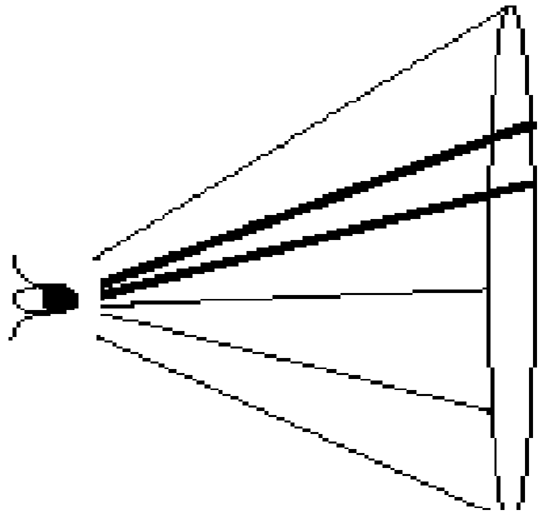
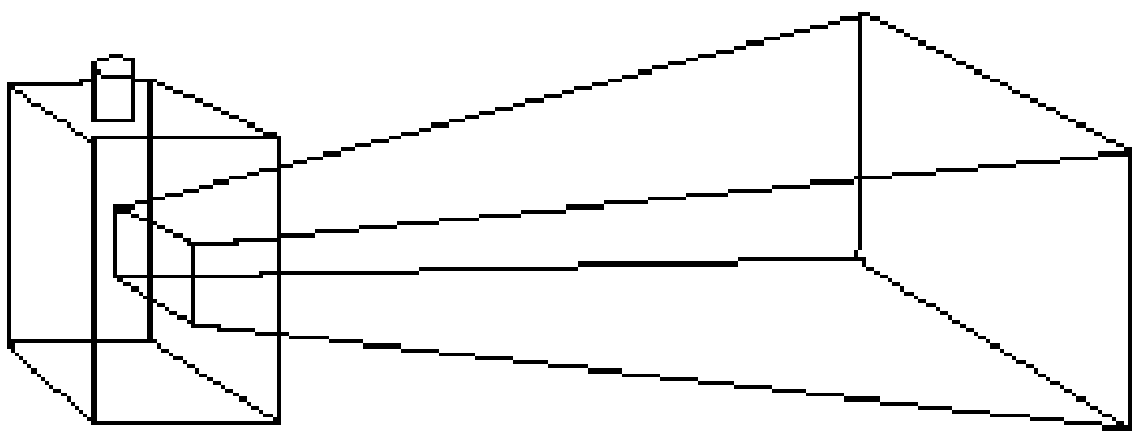
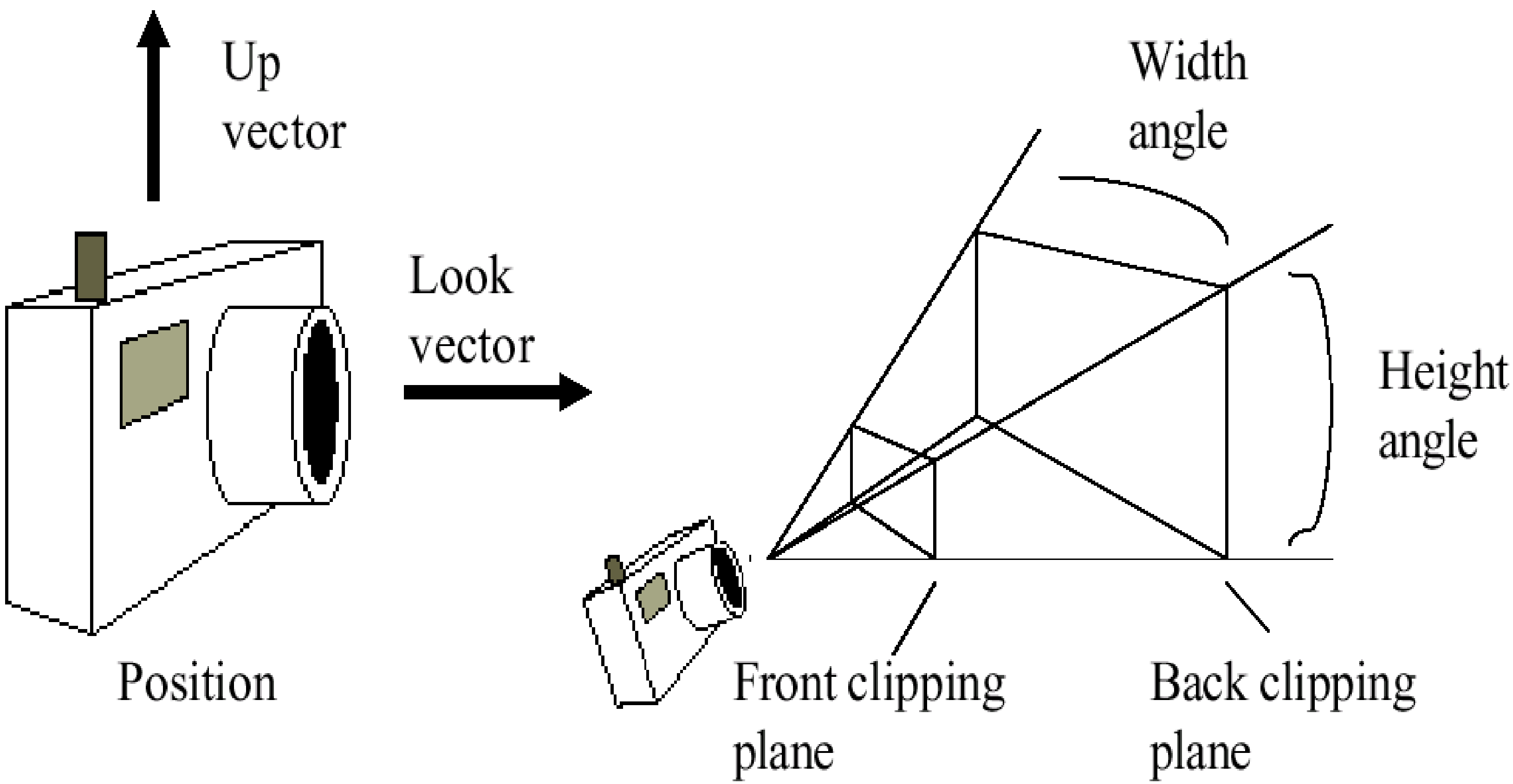
http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

3차원 View Volume

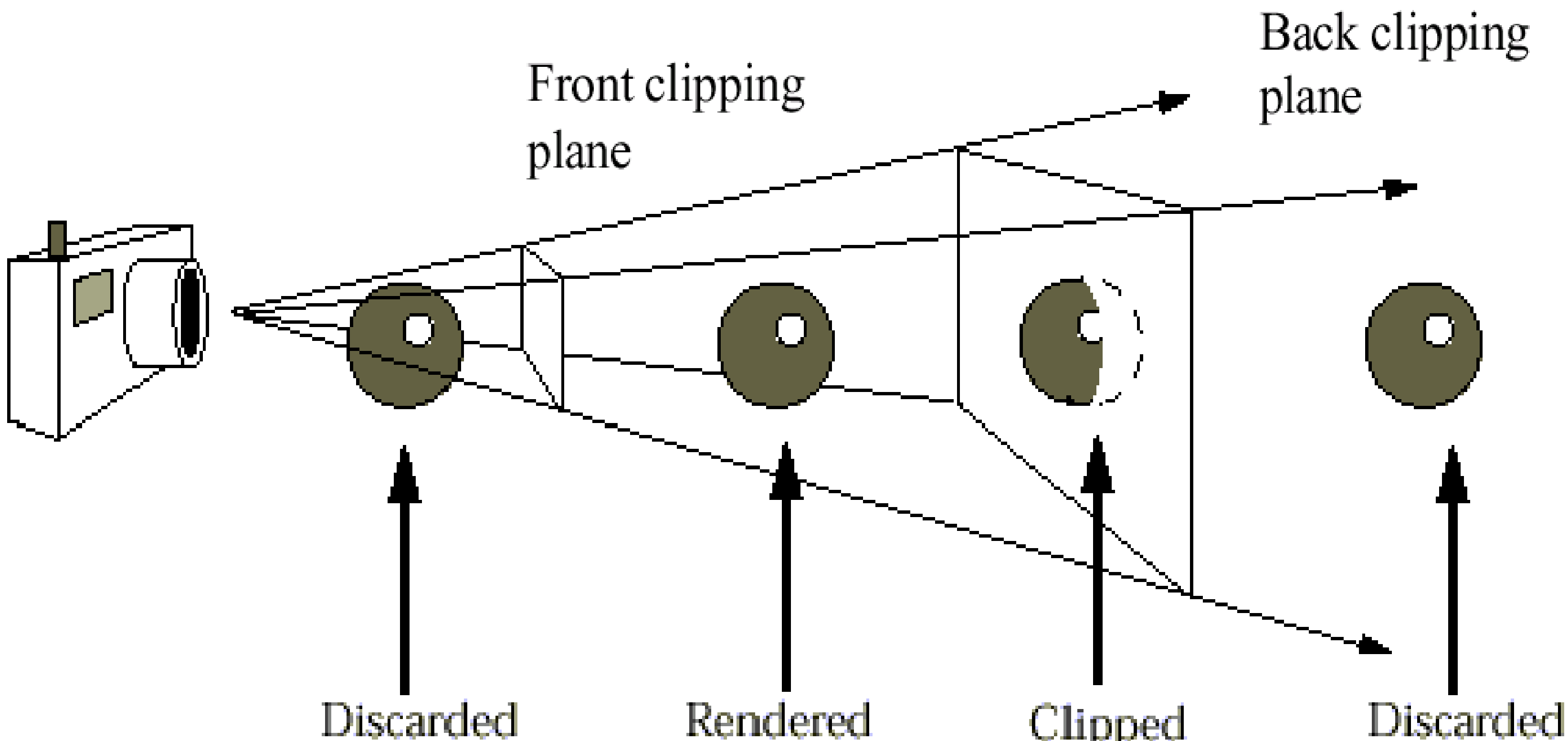




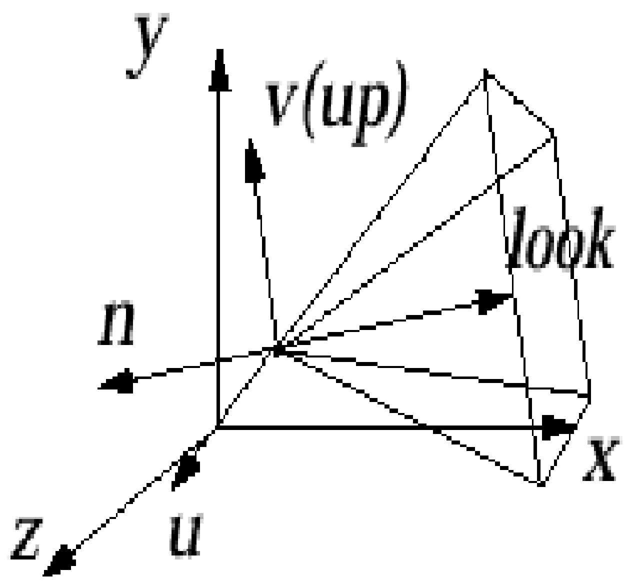
3차원 View Volume



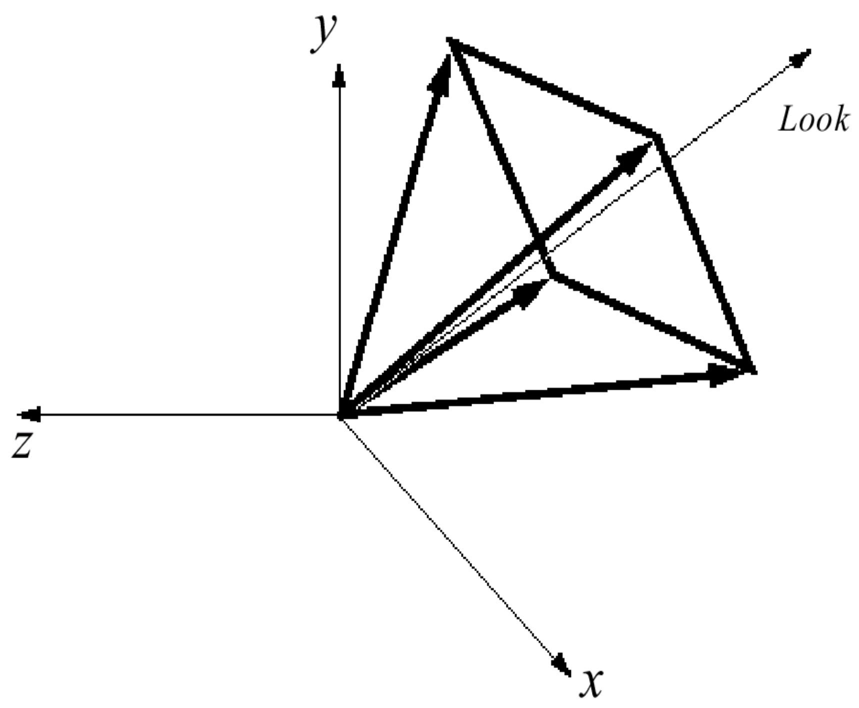
3차원 Clipping



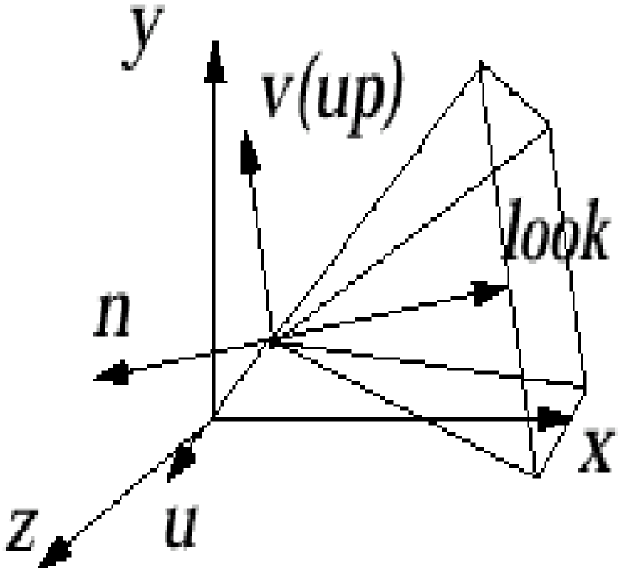
3차원 Viewing 변환



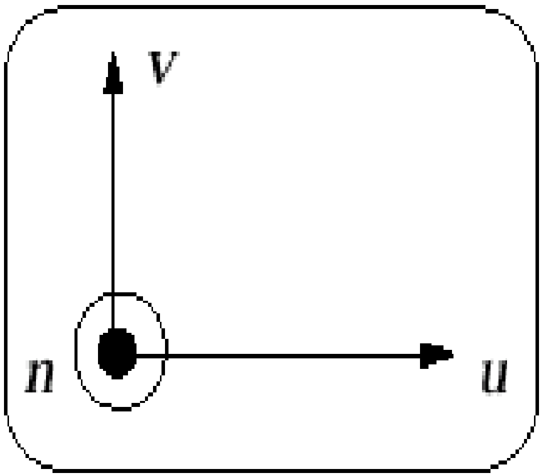
Arbitrary Perspective Frustum



3차원 Viewing 변환

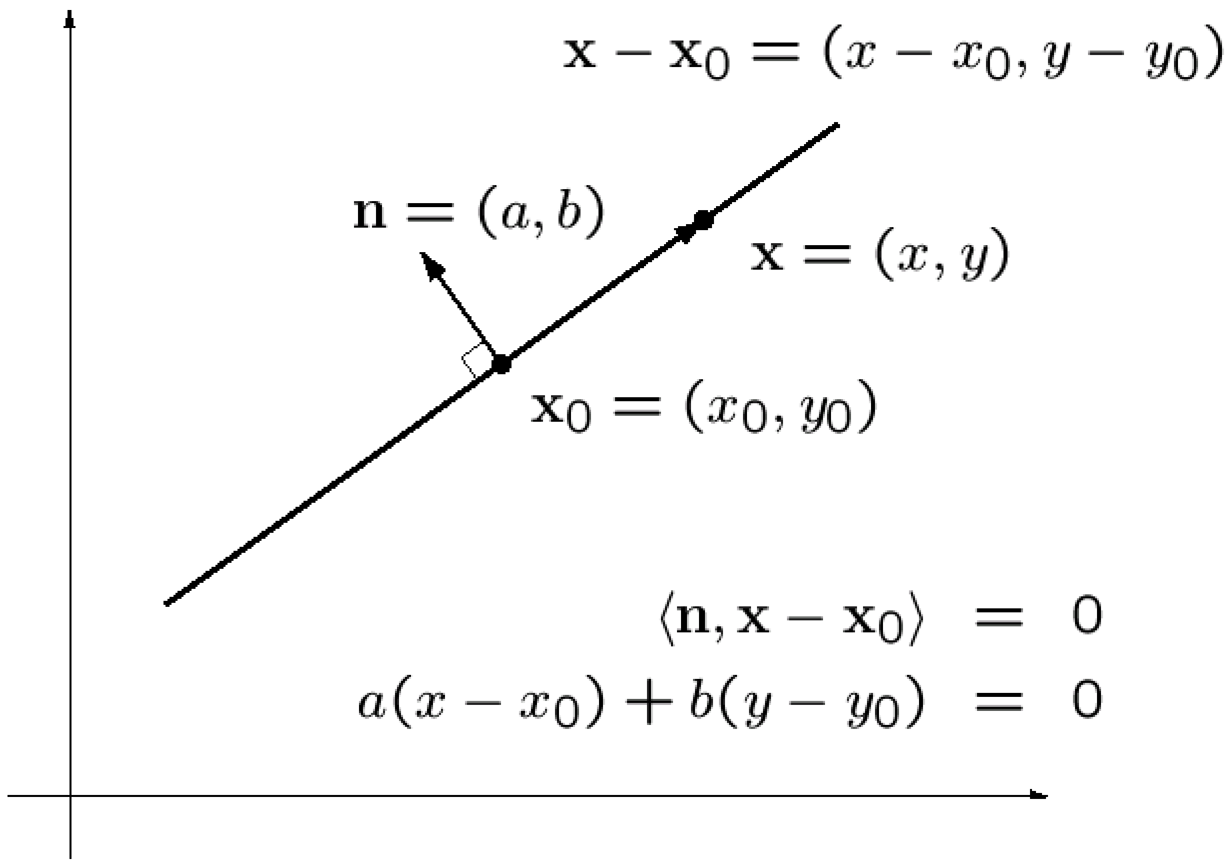


Arbitrary Perspective Frustum



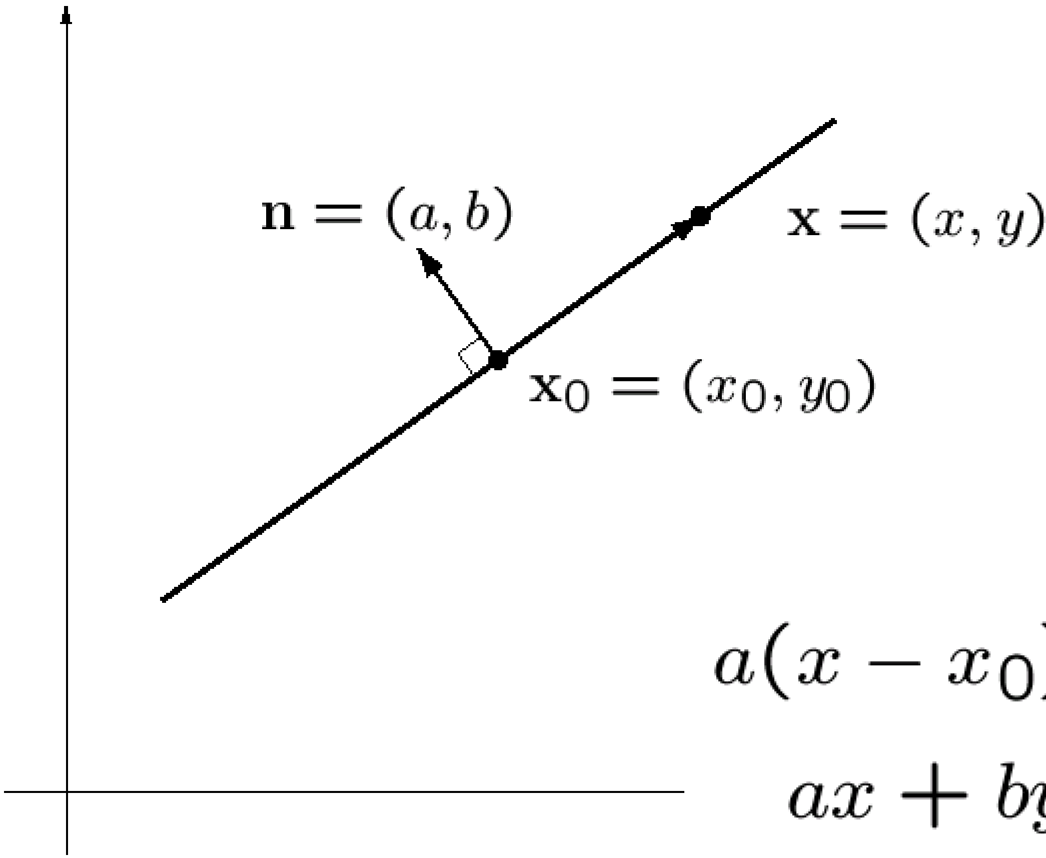
Camera Coordinate System
(with *n* coming out of the paper)

직선의 방정식



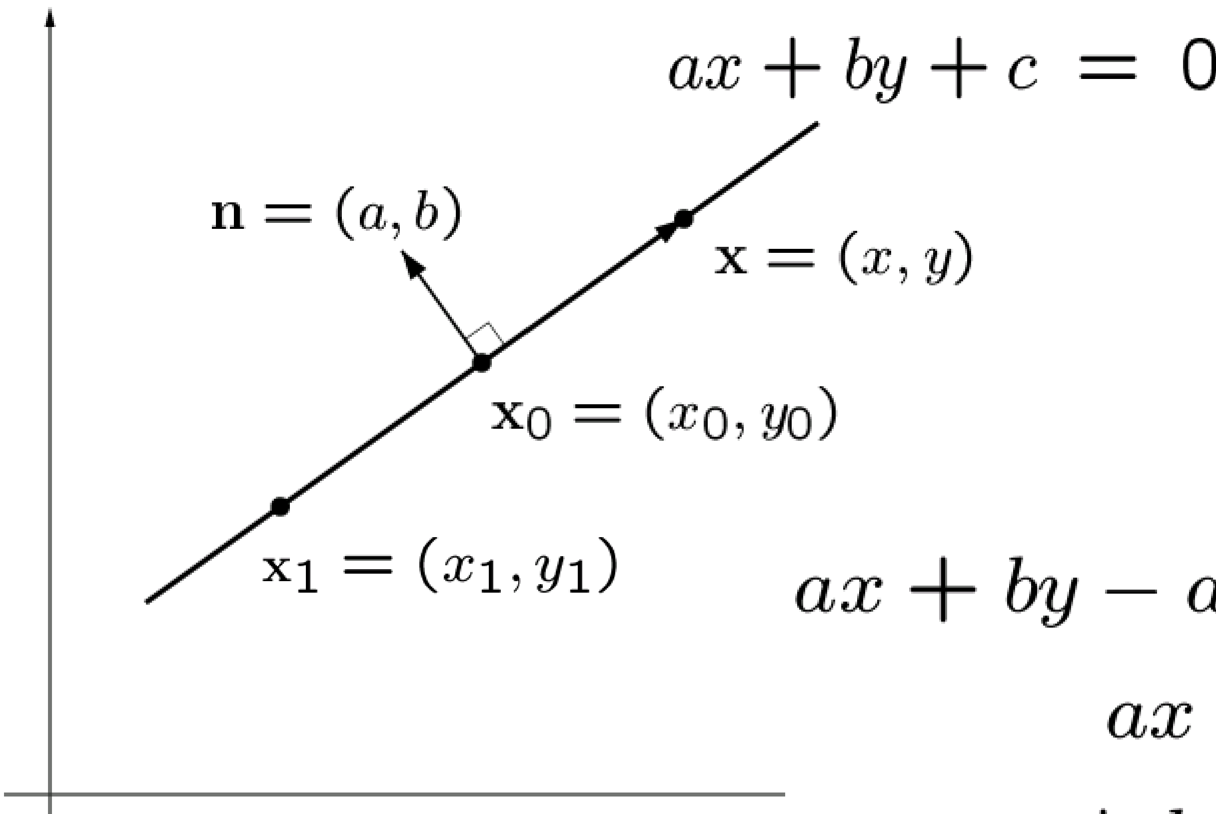
$$
\begin{aligned}
\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle &= 0 \\
a(x - x_0) + b(y - y_0) &= 0
\end{aligned}
$$

직선의 방정식



$$\begin{aligned} \langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle &= 0 \\ a(x - x_0) + b(y - y_0) &= 0 \\ ax + by - ax_0 - by_0 &= 0 \\ ax + by + c &= 0 \end{aligned}$$

직선의 방정식



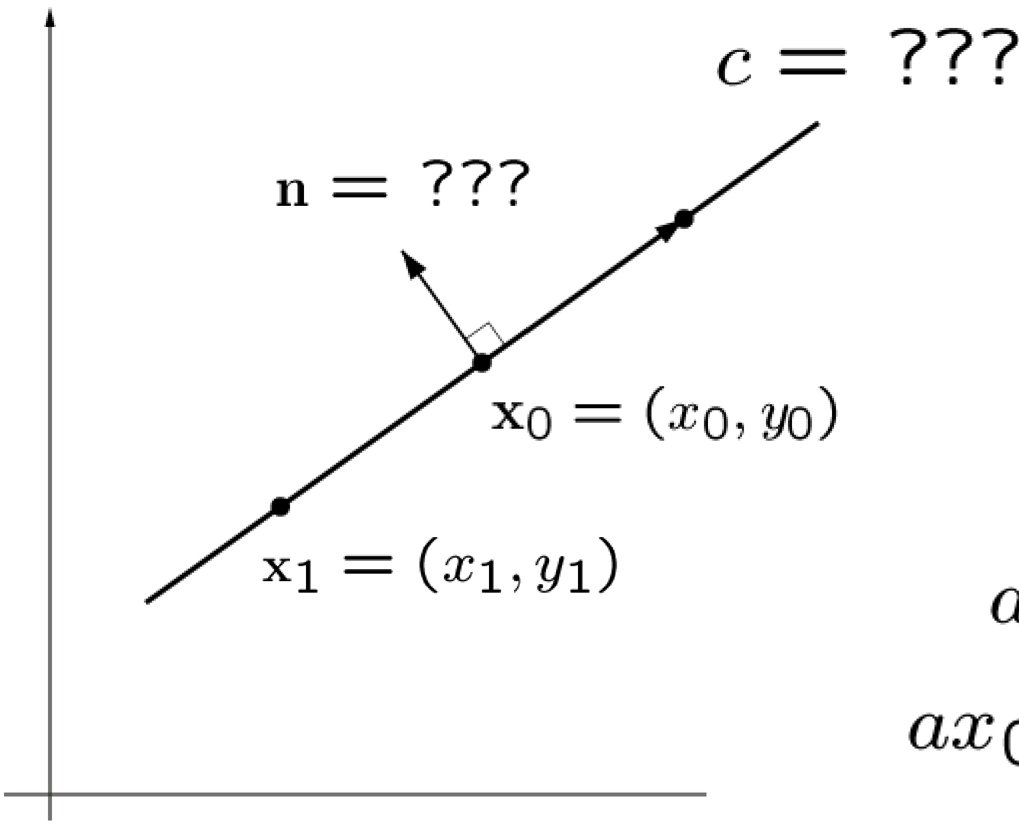
$$ax + by - ax_0 - by_0 = 0$$

$$ax + by + c = 0$$

$$ax_0 + by_0 + c \cdot 1 = 0$$

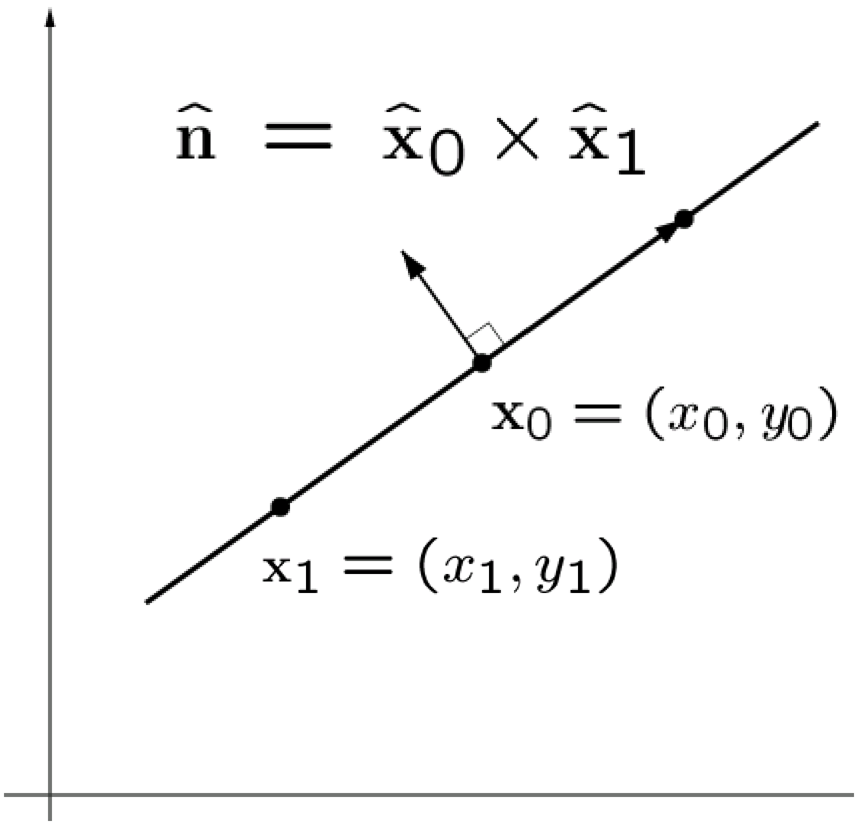
$$ax_1 + by_1 + c \cdot 1 = 0$$

두점으로 부터 직선구하기



$$
\begin{aligned}
ax + by + c \cdot 1 &= 0 \\
ax_0 + by_0 + c \cdot 1 &= 0 \\
ax_1 + by_1 + c \cdot 1 &= 0
\end{aligned}
$$

두점으로 부터 직선구하기



$$\hat{\mathbf{n}} = (a, b, c)$$

$$\hat{\mathbf{x}}_0 = (x_0, y_0, 1)$$

$$\hat{\mathbf{x}}_1 = (x_1, y_1, 1)$$

$$ax_0 + by_0 + c \cdot 1 = 0$$

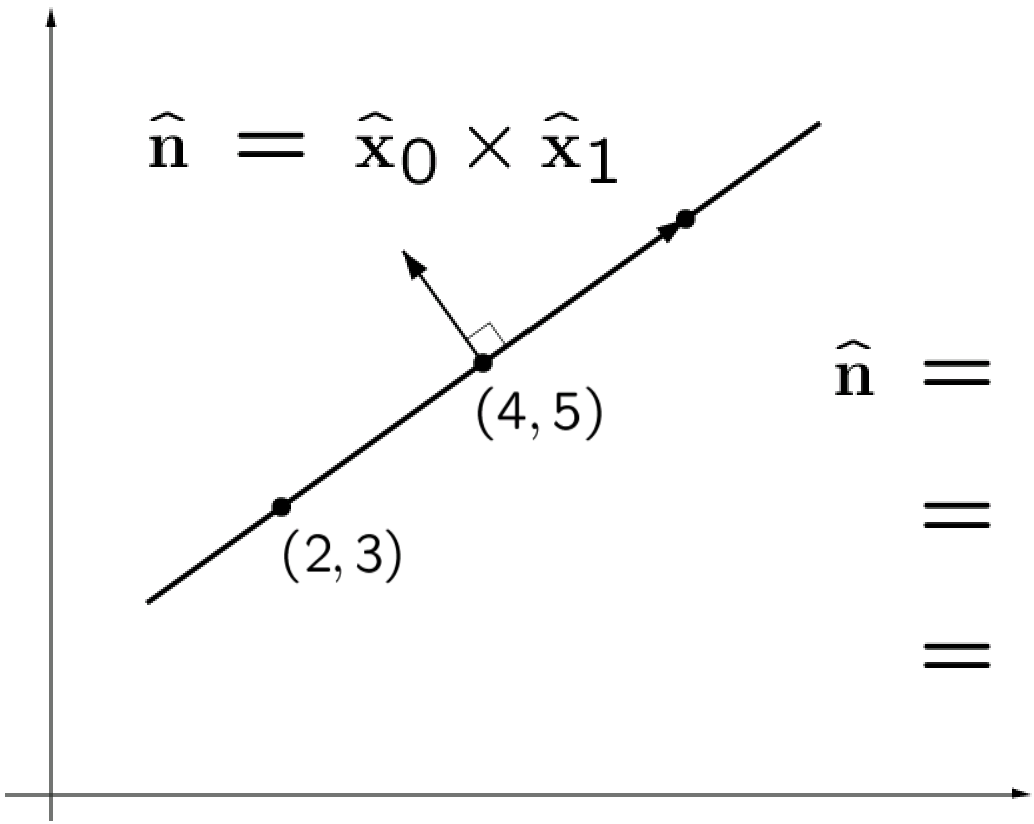
$$ax_1 + by_1 + c \cdot 1 = 0$$

$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_0 \rangle = 0$$

$$\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_1 \rangle = 0$$

두점으로 부터 직선구하기

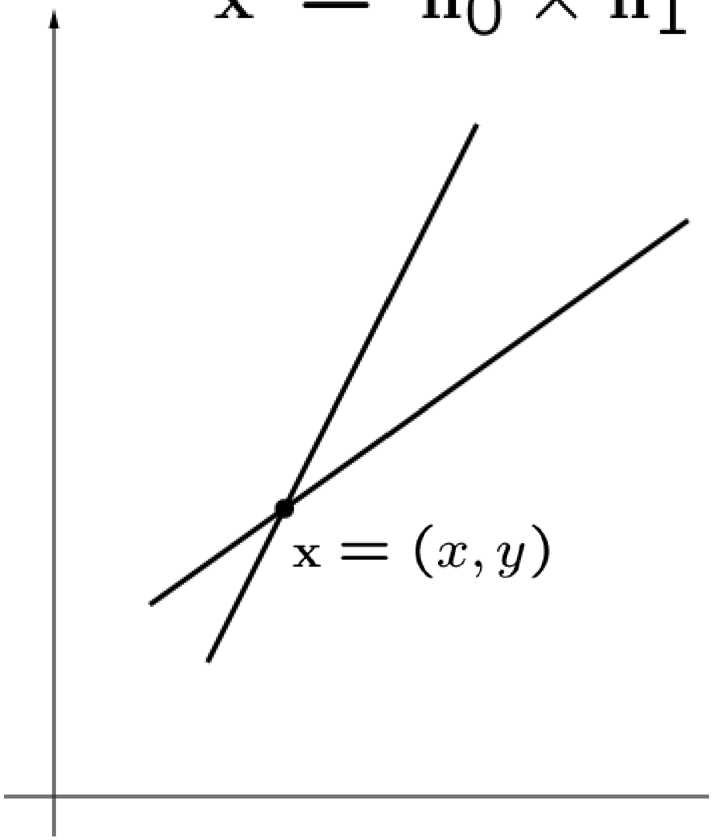
$$\hat{\mathbf{x}}_0 = (2,3,1)$$
$$\hat{\mathbf{x}}_1 = (4,5,1)$$



$$\begin{aligned}\hat{\mathbf{n}} &= \hat{\mathbf{x}}_0 \times \hat{\mathbf{x}}_1 \\ &= (2,3,1) \times (4,5,1) \\ &= (-2,2,-2)\end{aligned}$$

두직선으로 부터 교점구하기

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1$$



$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{n}}_0 = (a_0, b_0, c_0)$$

$$\hat{\mathbf{n}}_1 = (a_1, b_1, c_1)$$

$$a_0 \cdot x + b_0 \cdot y + c_0 \cdot 1 = 0$$

$$a_1 \cdot x + b_1 \cdot y + c_1 \cdot 1 = 0$$

$$\langle \hat{\mathbf{n}}_0, \hat{\mathbf{x}} \rangle = 0$$

$$\langle \hat{\mathbf{n}}_1, \hat{\mathbf{x}} \rangle = 0$$

두직선으로 부터 교점구하기

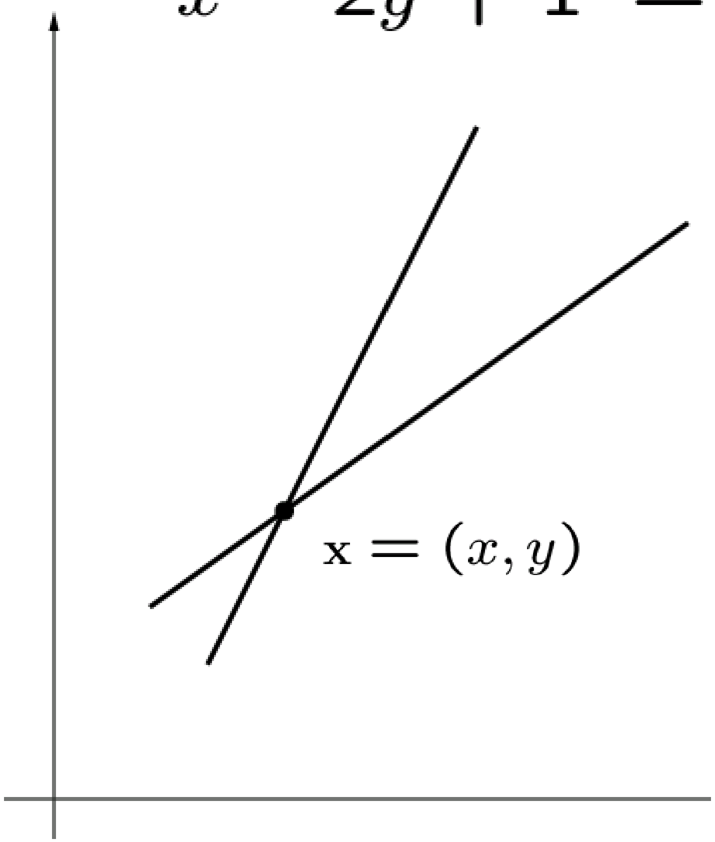
$$x + y - 1 = 0$$

$$x - 2y + 1 = 0$$

$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{n}}_0 = (1, 1, -1)$$

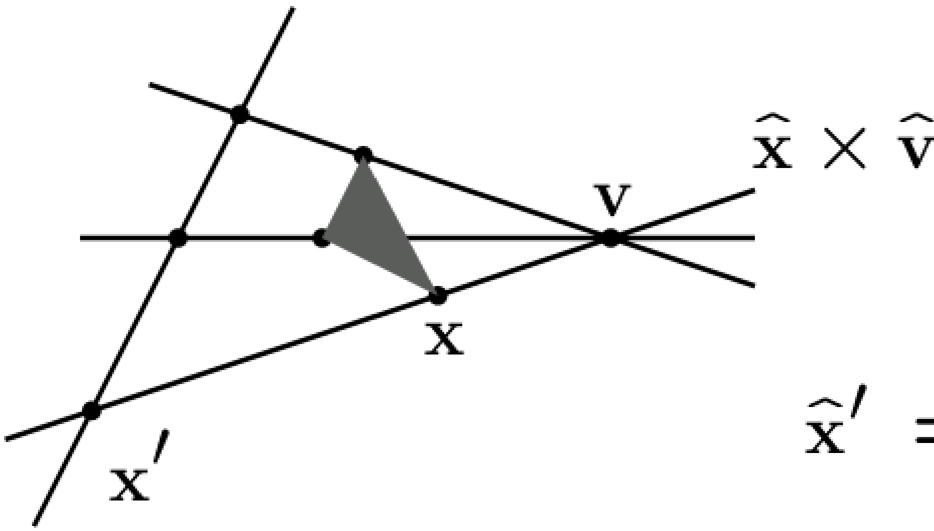
$$\hat{\mathbf{n}}_1 = (1, -2, 1)$$



$$\begin{aligned}\hat{\mathbf{x}} &= \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1 \\ &= (1, 1, -1) \times (1, -2, 1) \\ &= (-1, -2, -3) \\ &= \left(\frac{1}{3}, \frac{2}{3}, 1\right)\end{aligned}$$

2차원에서의 투영변환

$$\hat{\mathbf{n}} = (a, b, c)$$



$$\hat{\mathbf{x}} = (x, y, 1)$$

$$\hat{\mathbf{v}} = (v_x, v_y, 1)$$

$$\begin{aligned}
\hat{\mathbf{x}}' &= (x', y', 1) \\
&= \hat{\mathbf{n}} \times (\hat{\mathbf{x}} \times \hat{\mathbf{v}}) \\
&= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \, \hat{\mathbf{v}} \\
&= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \, \hat{\mathbf{x}} - \hat{\mathbf{v}} \, \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle
\end{aligned}$$

2차원에서의 투영변환

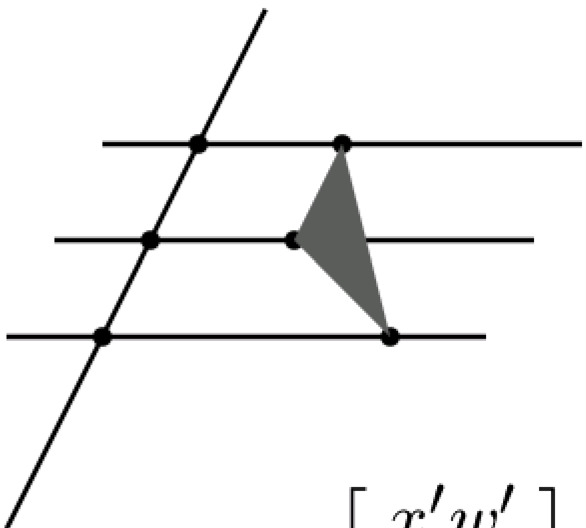
$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y + c & -bv_x & -cv_x \\ -av_y & av_x + c & -cv_y \\ -a & -b & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환



$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\leftarrow \quad \hat{\mathbf{v}} = (v_x, v_y, 0)$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} - \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y & -bv_x & -cv_x \\ -av_y & av_x & -cv_y \\ 0 & 0 & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3차원에서의 투영변환

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3차원에서의 평행투영변환

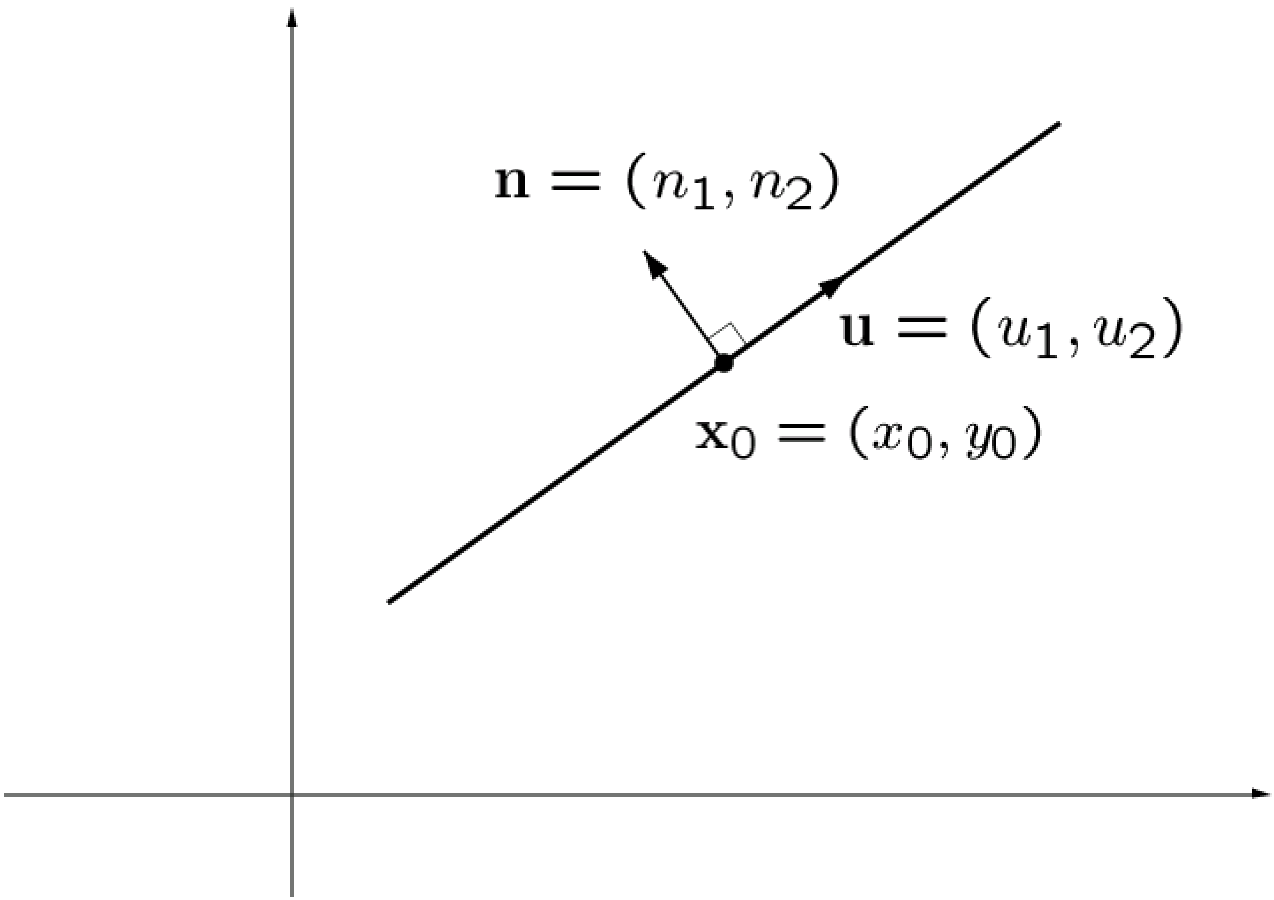
$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\|\mathbf{u}\| = \|\mathbf{n}\| = 1$$



$$\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

3차원에서의 Viewing 변환

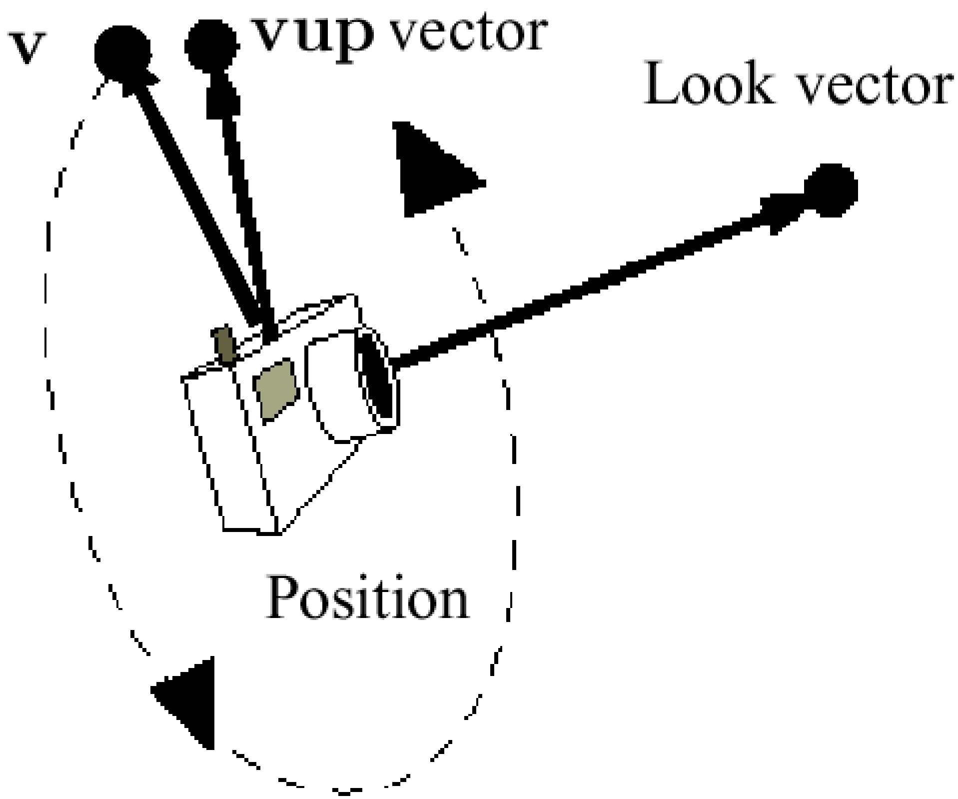
$$
\begin{aligned}
\mathbf{x} &= (x_0, y_0, z_0) \\
\mathbf{u} &= (u_1, u_2, u_3) \\
\mathbf{v} &= (v_1, v_2, v_3) \\
\mathbf{n} &= (n_1, n_2, n_3)
\end{aligned}
$$

$$\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{n} \rangle = \langle \mathbf{v}, \mathbf{n} \rangle = 0$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{n}\| = 1$$

$$
\begin{bmatrix}
u_1 & u_2 & u_3 & 0 \\
v_1 & v_2 & v_3 & 0 \\
n_1 & n_2 & n_3 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & -x_0 \\
0 & 1 & 0 & -y_0 \\
0 & 0 & 1 & -z_0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

3차원에서의 Viewing 변환



$$\mathbf{u} = \frac{\mathbf{vup} \times \mathbf{n}}{\|\mathbf{vup} \times \mathbf{n}\|}$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u}$$

3차원에서의 Viewing 변환

- 3차원 상의 점들을 2차원 viewing 평면으로 투영
- Viewing 평면상의 기준점 (x0,y0,z0)을 원점으로 이동
- U, V, N 방향을 X, Y, Z의 좌표계의 방향으로 회전
- 결과적으로 XY 평면상에 투영된 그림이 나타난다.